QUADRATIC FUNCTIONS: QUESTIONS AND ANSWERS

We usually write the defining equation for a quadratic function in one of two useful forms. These are:

Standard form: $f(x) = ax^2 + bx + c \quad (a \neq 0)$

Vertex form: $f(x) = a(x-h)^2 + k \quad (a \neq 0)$

We may ask a number of questions about a given quadratic function. The means used to obtain the answers often depend on the form in which the defining equation is written. Because of this, we shall place questions in the middle of the page, and provide most answers twice; once on the left side of the page (for use if the function is given in standard form), and once on the right side of the page (for use if the function is given in vertex form). Also, we shall use y and $f(x)$ interchangeably here, even though they technically have different meanings. Plotting the graph of a quadratic function always results in a non-linear curve whose shape is called a "parabola."

Q. How do I tell whether the parabola's arms open upward ("happy") or downward ("unhappy")? Also, how do I determine whether the parabola has a maximum or a minimum?
A. (Both forms) Examine the function; if $a > 0$, then the parabola is "happy" (technically, "concave up"), and if $a < 0$, then the parabola is "unhappy" ("concave down"). Concave-up parabolas always have a minimum, but never a maximum. Concave-down parabolas always have a maximum, but never a minimum. <u>Memory aid</u>: "*a* is for *attitude*:" "positive" attitude $\rightarrow$ "happy" parabola; "negative" attitude $\rightarrow$ "unhappy" parabola.

Q. How do I find the parabola's vertex? Is this related to the parabola's maximum/minimum?	
A. (Standard form) Calculate the *x*-coordinate using $x_{\text{vertex}} = \frac{-b}{2a}$. Then find the *y*-coordinate (y_{vertex}) by plugging x_{vertex} into the equation and solving for *y*. (Technically, find $f(x_{\text{vertex}})$ or $f\left(\frac{-b}{2a}\right)$.) The value of the maximum or minimum is equal to y_{vertex}.	**A.** (Vertex form) Find its coordinates by examining the function; $x_{\text{vertex}} = h$ and $y_{\text{vertex}} = k$. Be careful selecting the signs for *h* and *k*, since the formula contains a plus sign and a minus sign. The value of the maximum or minimum is equal to y_{vertex} or to *k*.

Q. How do I find the *x*-intercept(s) of the quadratic function's graph?	
A. (Standard form) Set *y* or $f(x)$ equal to zero, then solve the resulting quadratic equation for *x*, either by factoring or using the quadratic formula. If the discriminant $b^2 - 4ac$ is negative, then no *x*-intercepts exist (stop immediately if you see this).	**A.** (Vertex form) Set *y* or $f(x)$ equal to zero, then solve the resulting quadratic equation for *x*. Although one can multiply out the right side, it is usually easier to subtract *k* from both sides of the function, divide by *a*, take the $\pm$ square root, and then isolate *x*. If the inside of the square root is negative, then no *x*-intercepts exist (stop immediately if you see this).

Q. How do I find the *y*-intercept of the quadratic function's graph?	
A. (Standard form) Set *x* equal to zero, then solve the resulting expression for *y*. (Technically, find $f(0)$.) <u>Shortcut alternative</u>: Examine the function; the *y*-intercept is numerically equal to the value of *c*.	**A.** (Vertex form) Set *x* equal to zero, then solve the resulting expression for *y*. (Technically, find $f(0)$.) Be sure to follow order of operations correctly; in particular, compute $(x-h)^2$ *before* multiplying by *a*.

Q. How do I find the domain and range of the quadratic function?
A. (Both forms) The domain is always the set of all real numbers ($\mathbb{R}$), except when it is limited to a smaller region of the *x*-axis by additional wording given in a problem (this typically happens in word problems). If the parabola is concave-up, then the range is always $[y_{\text{vertex}}, \infty)$. If the parabola is concave-down, then the range is always $(-\infty, y_{\text{vertex}}]$. To determine y_{vertex}, see "How do I find the parabola's vertex?" above.

Q. How do I convert the function's definition to the other form (standard form to vertex form or vice-versa)?	
A. (Standard form $\rightarrow$ vertex form) Rewrite the function by completing the square (can be difficult).	**A.** (Vertex form $\rightarrow$ standard form) Multiply out the right side of the function's definition (is usually easy).