ESSENTIAL TRIGONOMETRIC IDENTITIES FOR PHYSICS & CALCULUS

For each real x, θ, α, and β that are elements of the domain of the specified functions, the following identities hold:

1. Definitions of derived trigonometric functions:

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta};\quad \cot\theta \equiv \frac{\cos\theta}{\sin\theta} \equiv \frac{1}{\tan\theta};\quad \sec\theta \equiv \frac{1}{\cos\theta};\quad \csc\theta \equiv \frac{1}{\sin\theta}$$

2. Even/odd properties of trigonometric functions:

$$\cos(-\theta) \equiv \cos(\theta);\qquad \sin(-\theta) \equiv -\sin(\theta);\qquad \tan(-\theta) \equiv -\tan(\theta)$$
$$\cot(-\theta) \equiv -\cot(\theta);\qquad \sec(-\theta) \equiv \sec(\theta);\qquad \csc(-\theta) \equiv -\csc(\theta)$$

3. Identities obtained from $\sin^2\theta + \cos^2\theta = 1$:

$$\tan^2\theta + 1 \equiv \sec^2\theta \qquad \cot^2\theta + 1 \equiv \csc^2\theta$$

4. Sums or differences of angles:

$$\cos(\alpha \pm \beta) \equiv \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\sin(\alpha \pm \beta) \equiv \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$
$$\tan(\alpha \pm \beta) \equiv \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \cot(\alpha \pm \beta) \equiv \frac{\cot\alpha\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$$
$$\sec(\alpha \pm \beta) \equiv \frac{\sec\alpha\sec\beta}{1 \mp \tan\alpha\tan\beta} \qquad \csc(\alpha \pm \beta) \equiv \frac{\csc\alpha\csc\beta}{\cot\beta \pm \cot\alpha}$$

5. Double angle formulas:

$$\cos(2\theta) \equiv \cos^2\theta - \sin^2\theta \equiv 2\cos^2\theta - 1 \equiv 1 - 2\sin^2\theta$$
$$\sin(2\theta) \equiv 2\sin\theta\cos\theta$$
$$\tan(2\theta) \equiv \frac{2\tan\theta}{1-\tan^2\theta} \qquad \cot(2\theta) \equiv \frac{\cot^2\theta - 1}{2\cot\theta}$$
$$\sec(2\theta) \equiv \frac{\sec^2\theta}{1-\tan^2\theta} \qquad \csc(2\theta) \equiv \frac{\csc^2\theta}{2\cot\theta}$$

6. Sum-to-product/difference-to-product formulas:

$$\cos\alpha + \cos\beta \equiv 2\cos\left[\tfrac{1}{2}(\alpha+\beta)\right]\cos\left[\tfrac{1}{2}(\alpha-\beta)\right]$$
$$\cos\alpha - \cos\beta \equiv 2\sin\left[\tfrac{1}{2}(\beta-\alpha)\right]\sin\left[\tfrac{1}{2}(\beta+\alpha)\right]$$
$$\sin\alpha \pm \sin\beta \equiv 2\sin\left[\tfrac{1}{2}(\alpha\pm\beta)\right]\cos\left[\tfrac{1}{2}(\alpha\mp\beta)\right]$$
$$\tan\alpha \pm \tan\beta \equiv \tan(\alpha\pm\beta)(1\mp\tan\alpha\tan\beta)$$

7. Product-to-sum formulas:

$$\cos\alpha\cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

8. Complementary angle and 90° rotation formulas:

$$\cos\left(\tfrac{\pi}{2}-\theta\right) \equiv \sin\theta \qquad \cos\left(\theta \pm \tfrac{\pi}{2}\right) \equiv \mp\sin\theta$$
$$\sin\left(\tfrac{\pi}{2}-\theta\right) \equiv \cos\theta \qquad \sin\left(\theta \pm \tfrac{\pi}{2}\right) \equiv \pm\cos\theta$$
$$\tan\left(\tfrac{\pi}{2}-\theta\right) \equiv \cot\theta \qquad \tan\left(\theta \pm \tfrac{\pi}{2}\right) \equiv -\cot\theta$$
$$\cot\left(\tfrac{\pi}{2}-\theta\right) \equiv \tan\theta \qquad \cot\left(\theta \pm \tfrac{\pi}{2}\right) \equiv -\tan\theta$$
$$\sec\left(\tfrac{\pi}{2}-\theta\right) \equiv \csc\theta \qquad \sec\left(\theta \pm \tfrac{\pi}{2}\right) \equiv \mp\csc\theta$$
$$\csc\left(\tfrac{\pi}{2}-\theta\right) \equiv \sec\theta \qquad \csc\left(\theta \pm \tfrac{\pi}{2}\right) \equiv \pm\sec\theta$$

9. Supplementary angle and 180° rotation formulas:

$$\cos(\pi-\theta) \equiv -\cos\theta \qquad \cos(\theta \pm \pi) \equiv -\cos\theta$$
$$\sin(\pi-\theta) \equiv \sin\theta \qquad \sin(\theta \pm \pi) \equiv -\sin\theta$$
$$\tan(\pi-\theta) \equiv -\tan\theta \qquad \tan(\theta \pm \pi) \equiv \tan\theta$$
$$\cot(\pi-\theta) \equiv -\cot\theta \qquad \cot(\theta \pm \pi) \equiv \cot\theta$$
$$\sec(\pi-\theta) \equiv -\sec\theta \qquad \sec(\theta \pm \pi) \equiv -\sec\theta$$
$$\csc(\pi-\theta) \equiv \csc\theta \qquad \csc(\theta \pm \pi) \equiv -\csc\theta$$

10. Squares of trigonometric functions:

$$\cos^2\theta \equiv \tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta \qquad \sin^2\theta \equiv \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta$$
$$\tan^2\theta \equiv \frac{1-\cos 2\theta}{1+\cos 2\theta} \qquad \cot^2\theta \equiv \frac{1+\cos 2\theta}{1-\cos 2\theta}$$
$$\sec^2\theta \equiv \frac{2}{1+\cos 2\theta} \qquad \csc^2\theta \equiv \frac{2}{1-\cos 2\theta}$$

11. Half angle formulas:

$$\cos\frac{\theta}{2} \equiv \pm\sqrt{\tfrac{1}{2}(1+\cos\theta)} \qquad \sin\frac{\theta}{2} \equiv \pm\sqrt{\tfrac{1}{2}(1-\cos\theta)}$$
$$\tan\frac{\theta}{2} \equiv \frac{\sin\theta}{1+\cos\theta} \equiv \frac{1-\cos\theta}{\sin\theta} \equiv \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

12. Compositions of trigonometric and inverse trigonometric functions (for $-1 \le x \le 1$ and $0 \le \theta \le \pi/2$):

$$\cos\left(\sin^{-1}x\right) \equiv \sqrt{1-x^2} \qquad \cos\left(\tan^{-1}x\right) \equiv \frac{1}{\sqrt{1+x^2}}$$
$$\sin\left(\cos^{-1}x\right) \equiv \sqrt{1-x^2} \qquad \sin\left(\tan^{-1}x\right) \equiv \frac{x}{\sqrt{1+x^2}}$$
$$\tan\left(\cos^{-1}x\right) \equiv \frac{\sqrt{1-x^2}}{x} \qquad \tan\left(\sin^{-1}x\right) \equiv \frac{x}{\sqrt{1-x^2}}$$
$$\cos^{-1}(\sin\theta) \equiv \frac{\pi}{2} - \theta \qquad \sin^{-1}(\cos\theta) \equiv \frac{\pi}{2} - \theta$$