BASIC ALGEBRA REVIEW

Assume that letters such as *a*, *b*, *x*, *y* represent real numbers, unless otherwise specified.

I. Subsets of the real numbers: Integers include whole numbers and their negatives. The rationals include all integers AND all fractions, and have repeating decimals. Irrationals (e.g., π, $\sqrt{2}$, *etc.*) include all other real numbers and have non-repeating decimals. A real number is either rational or irrational but not both.

II. Order: $a < b$ means that *a* is located to the left of *b* on the real number line; $a \le b$ means that either *a* is to the left of *b*, or *a* and *b* coincide. 0 is greater than all negative numbers, and less than all positive numbers.

III. Absolute value: $|a-b|$ is the distance (on number line) from *a* to *b*, disregarding direction (so the result of an absolute value is *always* non-negative). If $a > b$, then $|a-b| = a-b$; else $|a-b| = -(a-b) = b-a$. If $b = 0$, then $|a-0| = |a|$ is the distance from 0 to *a*. Watch negative signs closely; $|-3| = 3$ but $-|3| = -3$.

IV. Exponents: $a^0 = 1$ for $a \ne 0$. Change negative exponents to positive by exchanging the position of the associated factors from numerator to denominator or vice-versa; *e.g.*, $\dfrac{2^{-6}a^{-3}b^2}{c^4d^{-5}} = \dfrac{b^2d^5}{2^6a^3c^4}$ (note: do *not* exchange any factors that initially have positive exponents!). Other rules: $a^m \cdot a^n = a^{m+n}$; $\left(a^m\right)^n = a^{mn}$; $\dfrac{a^m}{a^n} = a^{m-n}$; $(ab)^m = a^m b^m$; $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$. Fractional exponents are really radicals: $a^{m/n} = \sqrt[n]{\left(a^m\right)} = \left(\sqrt[n]{a}\right)^m$.

V. Radicals (inverses of exponents): If $6^3 = 216$ then $\sqrt[3]{216} = 6$ (read "the cube root of 216 equals 6"; note that the same three numbers 6, 3, 216 appear in both equations, but in a different order). The small 3 above the radical is the *index*; the 216 inside the radical is the *radicand*. If the index is 2, it is usually not written; thus $\sqrt{25} = 5$. Simplify radicals when possible by factoring; for a cube root, any group of 3 identical factors is moved outside the radical and compressed into one factor; for fourth roots, any group of 4 identical factors is moved and compressed, *etc*.; *e.g.*, $\sqrt[3]{54x^4} = \sqrt[3]{\underline{2\cdot 2\cdot 2}\cdot 3\cdot \underline{x\cdot x\cdot x}\cdot x} = 2x\cdot\sqrt[3]{3x}$.

VI. Operations involving radicals: Product $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{a}$; quotient $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$; sum/difference $a\sqrt[n]{c} \pm b\sqrt[n]{c} = (a \pm b)\sqrt[n]{c}$ (similar to combining like terms); rationalizing denominators (one term) $\dfrac{a}{\sqrt{b}} = \dfrac{a}{\sqrt{b}}\cdot\dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{a\sqrt{b}}{b}$; rationalizing denominators (two terms, using conjugate of denominator) $\dfrac{a}{b\pm\sqrt{c}} = \dfrac{a}{b\pm\sqrt{c}}\cdot\dfrac{b\mp\sqrt{c}}{b\mp\sqrt{c}} = \dfrac{a\left(b\mp\sqrt{c}\right)}{b^2-c}$ or $\dfrac{a}{\sqrt{b}\pm\sqrt{c}} = \dfrac{a}{\sqrt{b}\pm\sqrt{c}}\cdot\dfrac{\sqrt{b}\mp\sqrt{c}}{\sqrt{b}\mp\sqrt{c}} = \dfrac{a\left(\sqrt{b}\mp\sqrt{c}\right)}{b-c}$; if an expression combines different indices (*e.g.*, a cube root times a fourth root), convert radicals to fractional exponents and use exponent rules to simplify (see **IV** above); result may be converted back to radicals afterwards.

VII. Factoring polynomials: Simplify like terms (if any) and rewrite in descending-powers order. Then try methods *in the order listed*: (1) GCF; (2) grouping (for four or more terms); (3) difference of squares, or sum/difference of cubes (for certain binomials); and (4) trinomial ("*ac*") method. It may be necessary to apply multiple methods (or repeatedly apply a single method) within a single expression, *e.g.*, $x^4 - 16 = \left(x^2+4\right)\left(x^2-4\right) = \left(x^2+4\right)(x+2)(x-2)$; the difference-of-squares method applies twice.